Introduction to Higher-Order Algebra for Level 1 and Level 2 Students

A Workshop by GED Testing Service®

GED TESTING SERVICE®

2

GED TESTING SERVICE®

Session Objectives

- Determine the big ideas of algebra
- Discuss the importance of teaching the basics of inequalities and functions (two High Impact Indicators)
- Connect inequalities and functions to real-world situations
- Share resources

3

The Magic of Algebra (What's the reason?)

Think of any number.

- Multiply it by 2.
- Add 4.
- Multiply by 3.
- Divide by 6.
- Subtract the number with which you started.

You got 2!

Explain with algebra why this works.

4

The answer is . . .

Start with the expression that describes the operations to be performed on your chosen number, *x*:

$$\frac{(2x + 4) \cdot 3 - x}{6}$$

and simplify the expression. You'll end up with 2, regardless of the value of *x*.

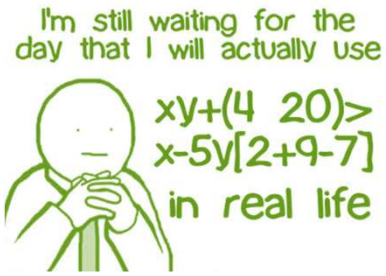
5

Why Use Magic Tricks or Puzzles?

- They are
  - Fun
  - Non-threatening
  - Motivational
  - Engaging
- Students begin to use algebraic thinking without knowing that is what they are doing.

6

Some Big Ideas in Algebra

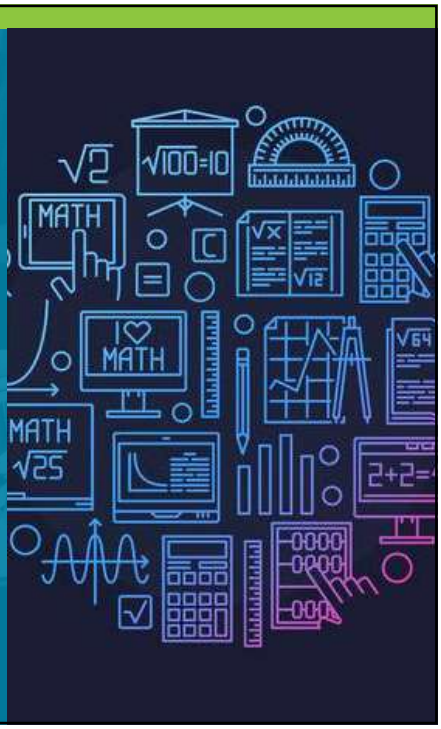


- Variable
- Symbolic Notation
- Equality
- Ratio and Proportion
- Pattern Generalization
- Equations and Inequalities
- Multiple Representations of Functions

7

Holding Algebra in Your Hands

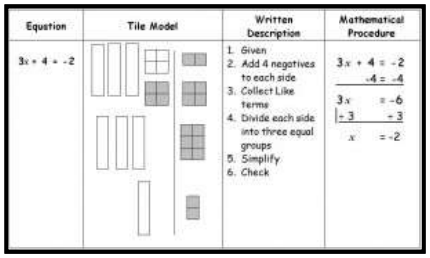
Starting with the concrete by using algebra tiles



8

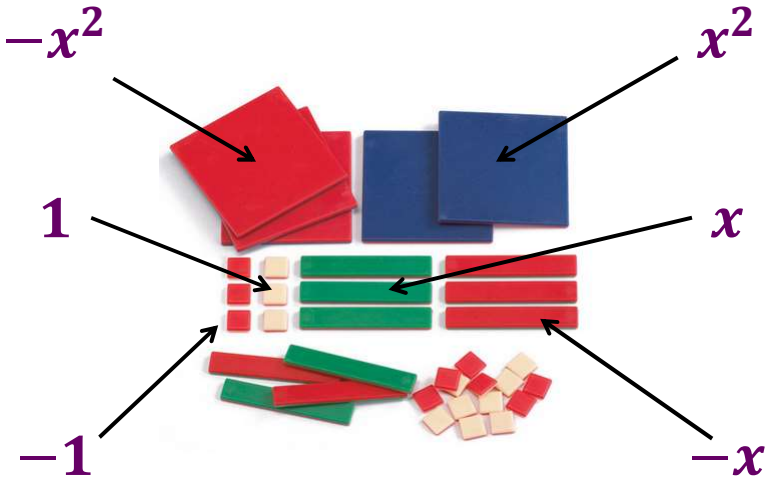
When teaching algebraic concepts, model using multiple representations

- Start with the concrete
- Represent problems using symbols, expressions, and equations, tables, and graphs
- Model real-world situations
- Complete problems different ways (flexibility in problem solving)



9

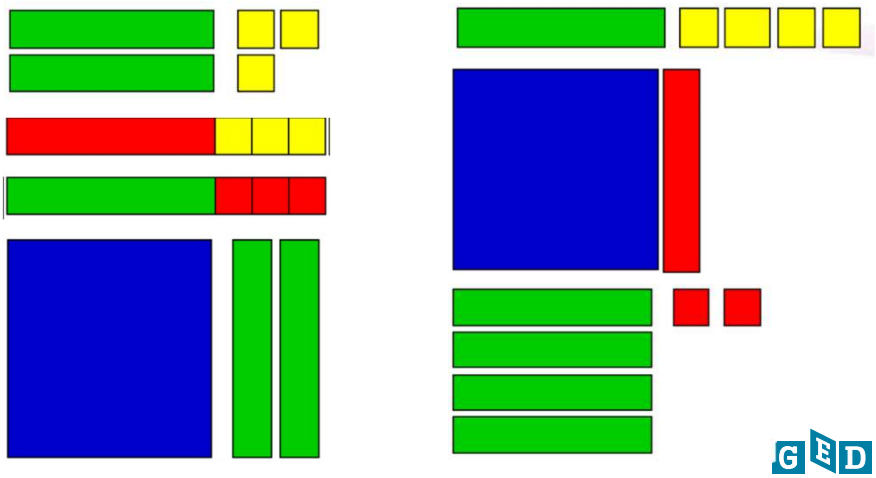
Introduction to Algebra Tiles



Remember, they could be called x, y, b, t, etc.

10

What's My Polynomial?



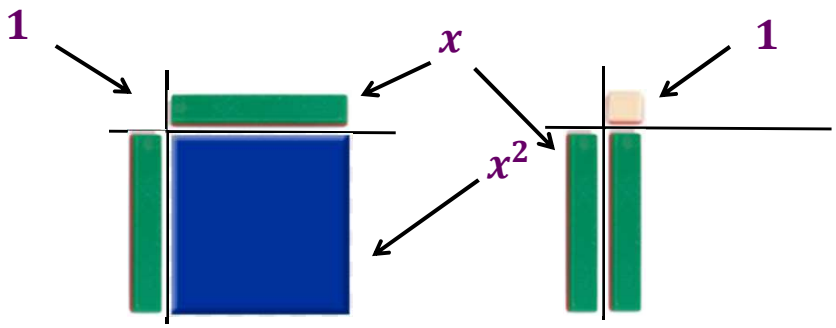
11

Big Ideas Using Algebra Tiles

- Adding and Subtracting Integers; Zero Principle
- Modeling Linear Expressions
- Solving Linear Equations
- Simplifying Polynomials
- Solving Equations for Unknown Variable
- Multiplying and Dividing Polynomials
- Factoring Trinomials
- Completing the Square
- Investigations

12

Look at the Relationships among the Tiles



13

Let's Use Algebra Tiles to Multiply Polynomials

Problem: (x + 2)(x + 3) = ?

Step 1: Gather all the tiles you need for each term.

x + 2 

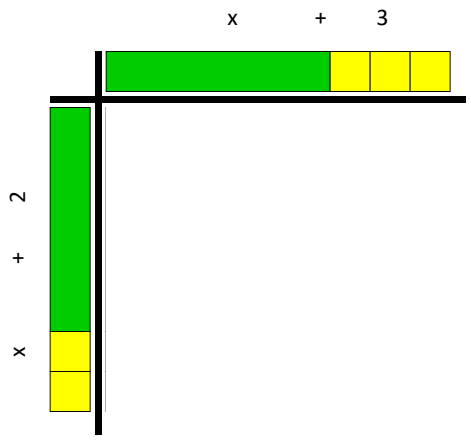
x + 3 

14

Multiplying Polynomials

$(x + 2)(x + 3) = ?$

Step 2: Lay out the tiles on opposite axes of the algebra tile mat as shown below.

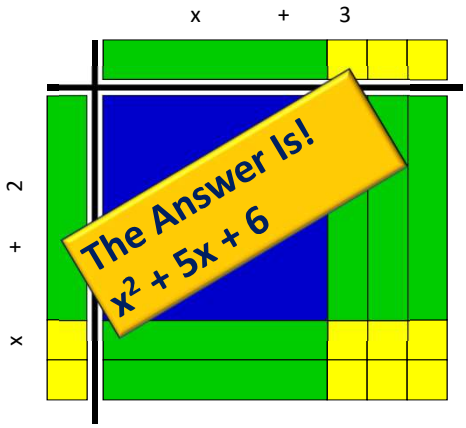


15

Multiplying Polynomials

$(x + 2)(x + 3) = ?$

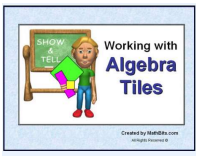
Step 3: Using other tiles, form a perfect rectangle using the tiles on axes of the product mat as the indicators for the dimensions of the rectangle as shown.



Notice that no other combination of tiles would make a perfect rectangle matching the dimension of tiles on the axes of the product mat.

16

Resources



- Working With Algebra Tiles – MathBits
  http://mathbits.com/MathBits/AlgebraTiles/AlgebraTiles.htm
- Factoring Polynomials Using Algebra Tiles - Del Mar College
  http://dmc122011.delmar.edu/math/MLC/QEPMathSeminars/FactoringTrinomialsAlgebraTilesStudentActivity.pdf
- Multiplying Polynomials Using Algebra Tiles – Virginia Dept. of Education
  http://www.doe.virginia.gov/testing/solsearch/sol/math/A/mess_a-2b_2.pdf

17

Algebra Tile Apps

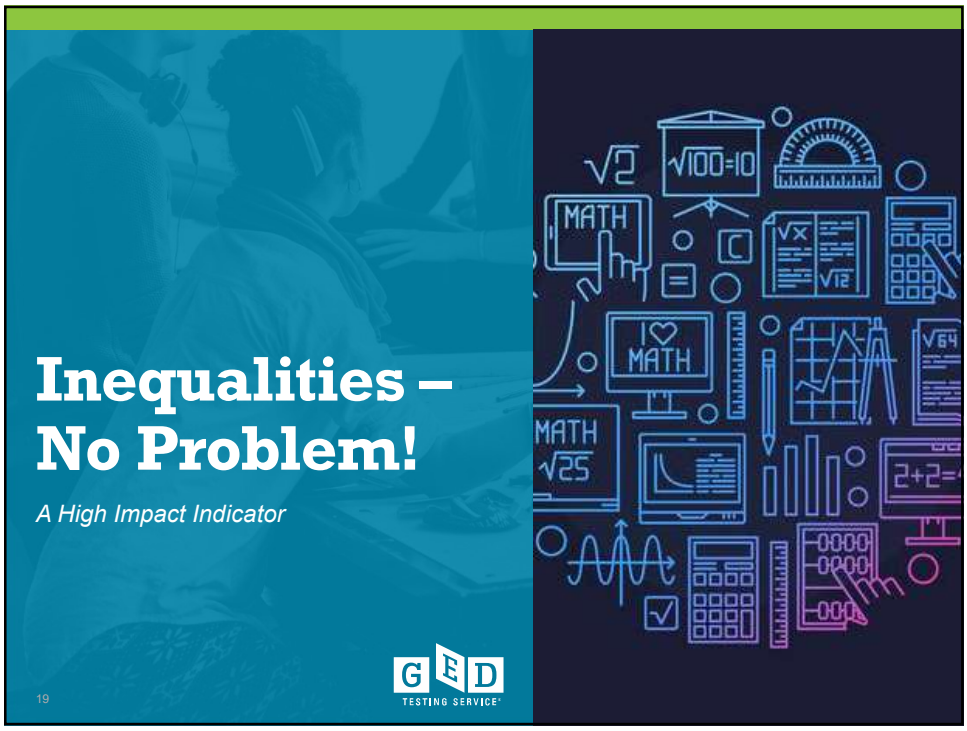
- Illuminations (National Council for Teachers of Math)
  http://illuminations.nctm.org/activity.aspx?id=3482

- Michigan Virtual University
  http://media.mivu.org/mvu_pd/a4a/homework/index.html

- National Library of Virtual Manipulatives
  http://nlvm.usu.edu/en/nav/vlibrary.html

Don't forget phone apps!

18



Focusing on High Impact Indicators - Inequalities

A.3 Write, manipulate, solve, and graph linear inequalities

- **A.3.a** Solve linear inequalities in one variable with rational number coefficients.
- **A.3.b** Identify or graph the solution to a one variable linear inequality on a number line.
- **A.3.c** Solve real-world problems involving inequalities.
- **A.3.d** Write linear inequalities in one variable to represent context.

20

GED TESTING SERVICE

Can your students . . .

Solve inequalities in one variable, using the standard algorithms?

Solve a one-variable inequality and identify or create a graph on the number line of the solution?

Analyze the relationship between quantities in a real-world problem, and then create an inequality to model the problem situation?

Analyze the relationship between quantities in a real-world problem, and then solve the problem through algebraic reasoning?

21

What is an inequality?

An inequality is a mathematical sentence that uses symbols such as <, ≤, >, or ≥ to compare two quantities.

22

Inequalities Are Everywhere

Situation	Mathematical Inequality
Speed limit	Legal speed on the highway ≤ 65 miles per hour
Credit card	Monthly payment ≥ 10% of your balance in that billing cycle
Text messaging	Allowable number of text messages per month ≤ 250
Travel time	Time needed to drive from home to school/work ≥ 18 minutes

23

Recognize the Symbols and the Vocabulary

Term	Inequality
Coefficient	4a>8
Boundary Point	A solution that makes the inequality true
Solution Set	The range of values that make the inequality true
Inclusive	a ≤ 6
Exclusive	a < 6

24

Rules for Solving Inequalities

1. Make the same changes to both sides of the inequality
2. Isolate the variable
3. Combine like terms
4. Use the inverse operation to remove clutter from the variable
5. If your inverse operation is multiplication or division by a negative number, reverse the inequality sign

< becomes >
> becomes <
≤ becomes ≥
≥ becomes ≤

25

Properties of Inequalities

Addition and Subtraction

If a > b, then a + c > b + c

If a > b, then a - c > b – c

Real-life situation

Becky is older than Janet: b > j

Add 10 years: b + 10 > j + 10

Subtract 10 years: b – 10 > j - 10

26

Properties of Inequalities

Multiplication and Division

If a > b, then ac > bc, if c > 0

If a < b, then ac < bc, if c <0

Real-life situation

Becky is older than Janet: b > j

When they are twice their current age:

b(2) > j(2)

When they were half the age they are now:

$\frac{b}{2} > \frac{j}{2}$

27

But…there is one exception

$-3n > 12$

If you divide or multiply by a negative number

$\frac{-3n}{-3} > \frac{12}{-3}$

Reverse the inequality symbol

$n < -4$

Solution: all numbers less than - 4

28

Testing the Property

$3 > 2$

Multiply by -1

$(-1)(3) > 2(-1)$

$-3 > -2$ **FALSE**

$-3 < -2$ **TRUE**

Multiplying by a negative flipped the inequality sign from "greater than" to "less than."

29

Solve the Inequality

$4 + x < 12$

$4 + \boxed{x} < 12$ (draw wall down inequality)

$4 + x < 12$ (box in variable)

$4 + x < 12$ (minus 4 both sides)

$-4 \quad -4$

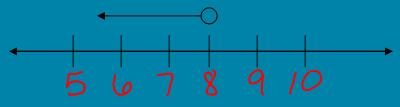
$x < 8$

30

Graph the Solution

$x < 8$

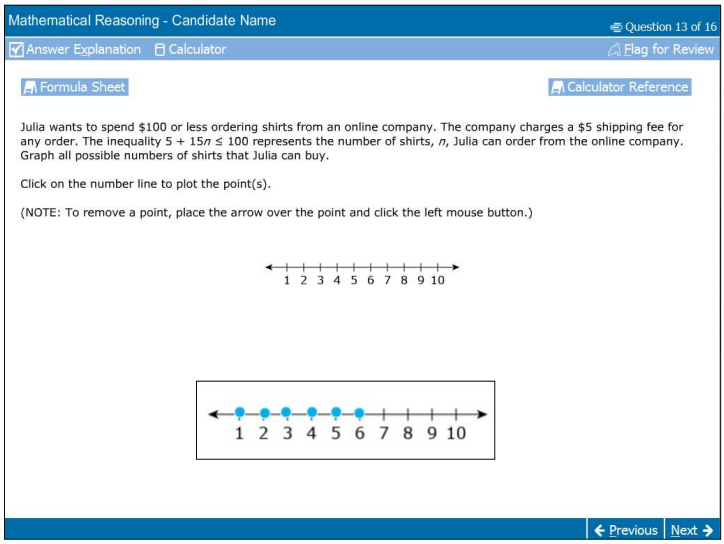
1. Draw a number line. Just need a few numbers on either side of the solution number.



2. Decide if open circle or closed circle. Place it above the solution number.
3. Determine which way your arrow goes by substituting a number in for the variable to make the statement true. Then draw the arrow pointing in that direction.

31

Sample Question from GEDTS



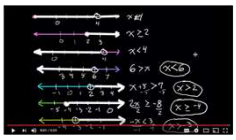
32

Resources

One-Variable Inequalities – Khan Academy

https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities

Virtual Nerds: What is an Inequality?
https://www.youtube.com/watch?v=wcBwdz-ZBaM



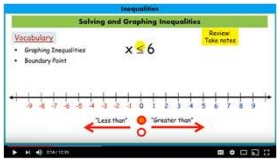
Very Basics of Graphing Inequalities (on a number line)
https://www.youtube.com/watch?v=nif2PKA9bXA

33

Resources

Math is Fun – Solving Inequalities

http://www.mathsisfun.com/algebra/inequality-solving.html



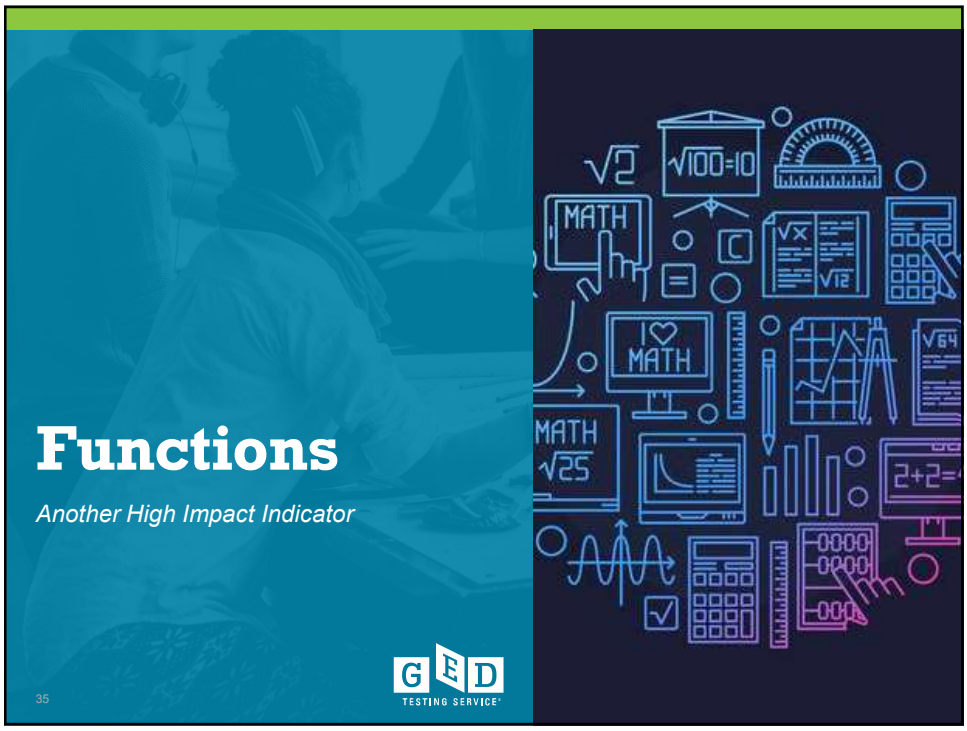
Solving and Graphing Inequalities (Excellent!)

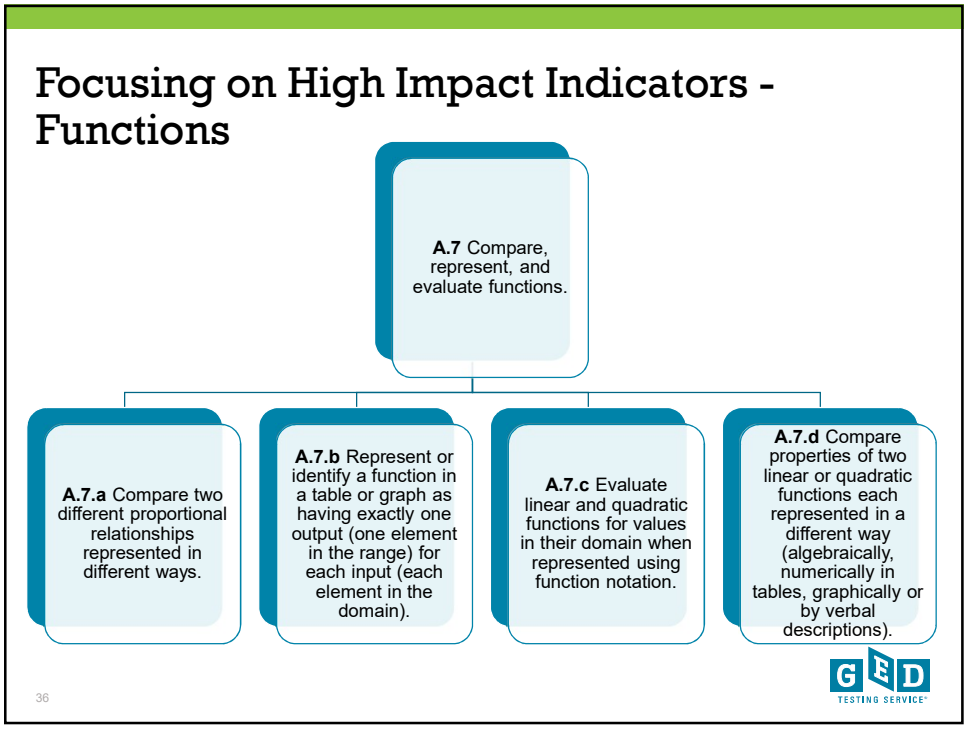
https://www.youtube.com/watch?v=EE2qWIyjKD0

Math Dude Unit 1-4 – Solving Inequalities

https://www.youtube.com/watch?v=8hhewFQ_K0w

34





Can your students . . .

Identify functions and non-functions displayed in graphs and tables, and create functions (graphs/tables)?

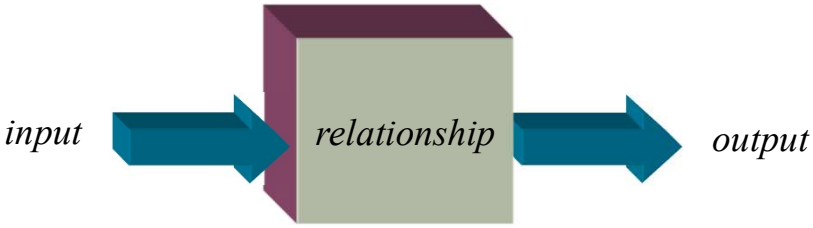
Substitute values for variables in functions and evaluate the resulting numerical expressions?

Convert functional representations from one from to another, and compare properties of the functions?

37

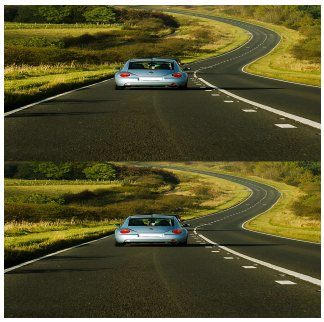
What is a function?

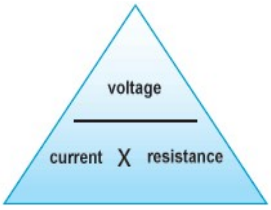
A **function** is a special relationship between a set of inputs and a set of permissible outputs. It is a useful mathematical tool.



38

Functions in Daily Life

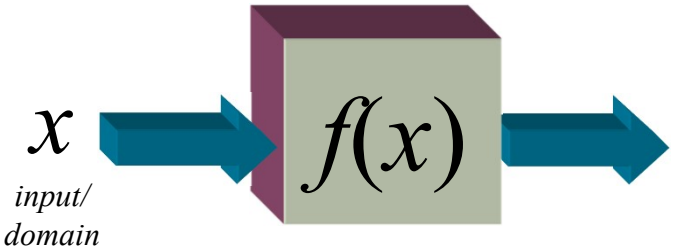




39

What is a function? Looking Closer

A **function** is a relation in which each element of the domain is paired with *exactly one* element of the range. Another way of saying it is that there is one and only one output (y) with each input (x).



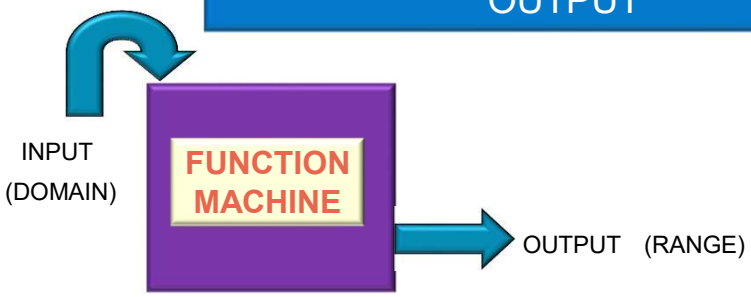
40

In order for a relationship to be a function . . .

EVERY INPUT MUST HAVE AN OUTPUT

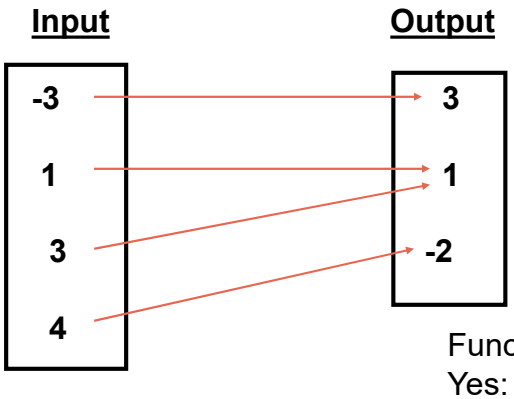
TWO DIFFERENT INPUTS CAN HAVE THE SAME OUTPUT

ONE INPUT CAN HAVE ONLY ONE OUTPUT



41

Is it a function?



Domain (input) = {-3, 1, 3, 4}
Range (output) = {3,1,-2}

Function?
Yes: each input is mapped onto exactly one output

42

Real World – Is It a Function?

People and Social Security Numbers

Domain	All people with a valid social security number
Range	All valid social security numbers
Is it a function?	Yes

43

Real World – Is It a Function?

People and Phone Numbers

Domain	All people who have a phone
Range	Phone numbers of all people who have a phone
Is it a function?	No

44

Show Functions Four Ways

x	y
-4	-11
-2	-7
0	-3
2	1
4	5

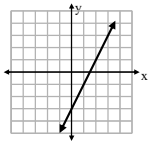
Table

$y = 2x - 3$

Equation

y is 3 less than twice a number x

Written Description



Graph

45

Vocabulary

x	*y*
Input	Output
Domain	Range
Independent Variable	Dependent Variable
x	f(*x*)

46

Is it a function? Creating Input/Output Tables

{(4,3), (-2, 10), (4, -6), (10,7)}

There are two inputs that are the same number, but each has a different output. A relationship does not exist, so no, it is not a function.

Input x	*Output y*
4	3
-2	10
4	-6
10	7

47

Is it a function?

Which of the following relations are functions?

R= {(9,10), (-5, -2), (2, -1), (3, -9)}

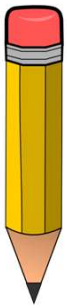
S= {(6, a), (8, f), (6, b), (-2, p)}

T= {(z, 7), (y, -5), (r, 7) (z, 0), (k, 0)}

48

Is it a function?

Vertical Line Test: *a relation is a function if a vertical line drawn through its graph, passes through only one point.*

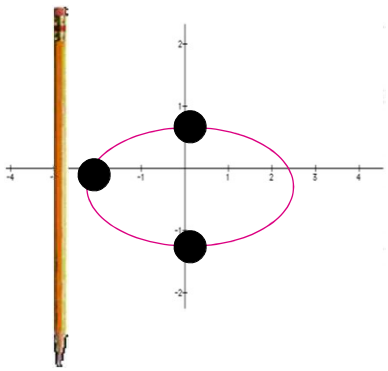


AKA: ***"The Pencil Test"***

Lay the pencil perpendicular to the x-axis. Move the pencil ***left to right (–x to x)****; if it crosses more than one point, it is not a function.*

49

Vertical Line Test

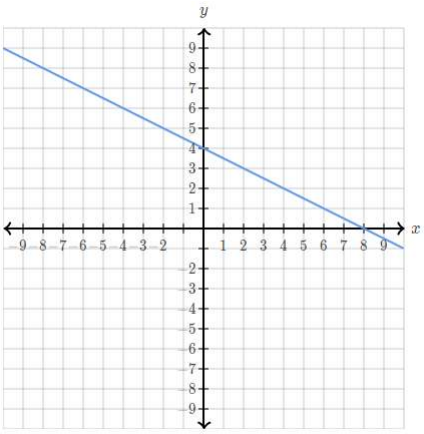


Would this graph be a function?

NO

50

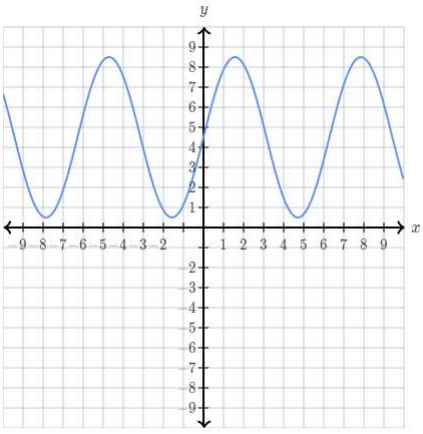
Is it a function?



Yes

51

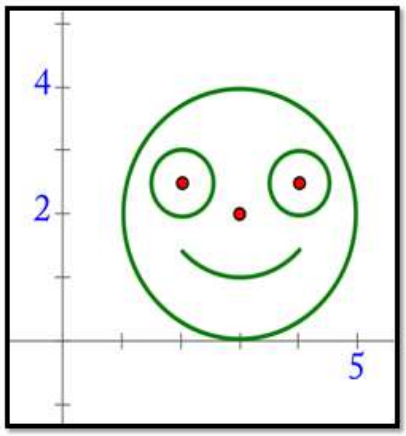
Is it a function?



Yes

52

Is it a function?



No

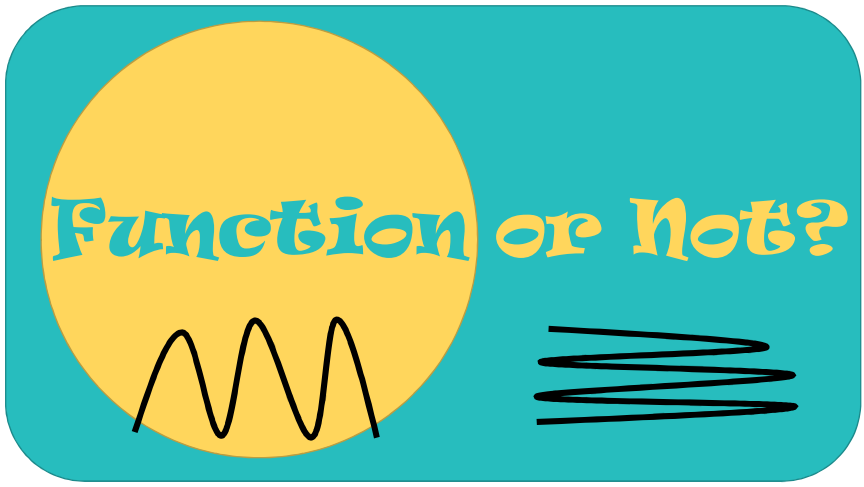
53

Back to Real-World Situations

Is it a function?

- The relation of distance and time during a trip.
- The relation of a month to the length of daylight.
- The relation of a person's shoe size to their height.
- The relation of amount of money earned and hours worked.

54



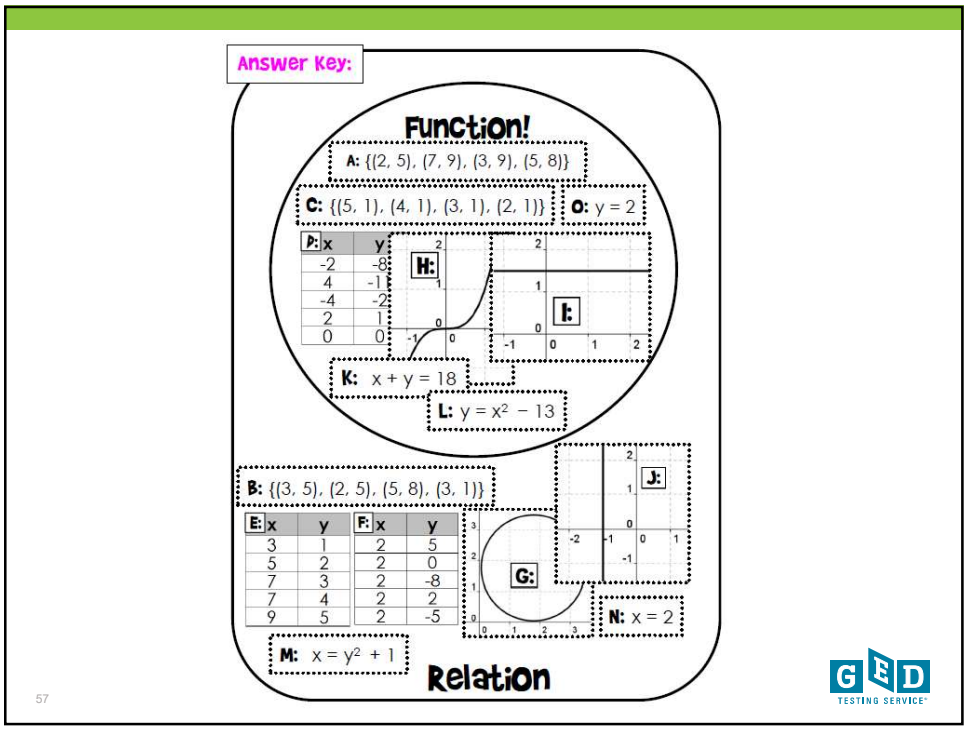
A free activity from Scaffolded Math and Science at Teachers Pay Teachers
https://www.teacherspayteachers.com/Product/Function-or-Not-Sorting-Activity-1699717

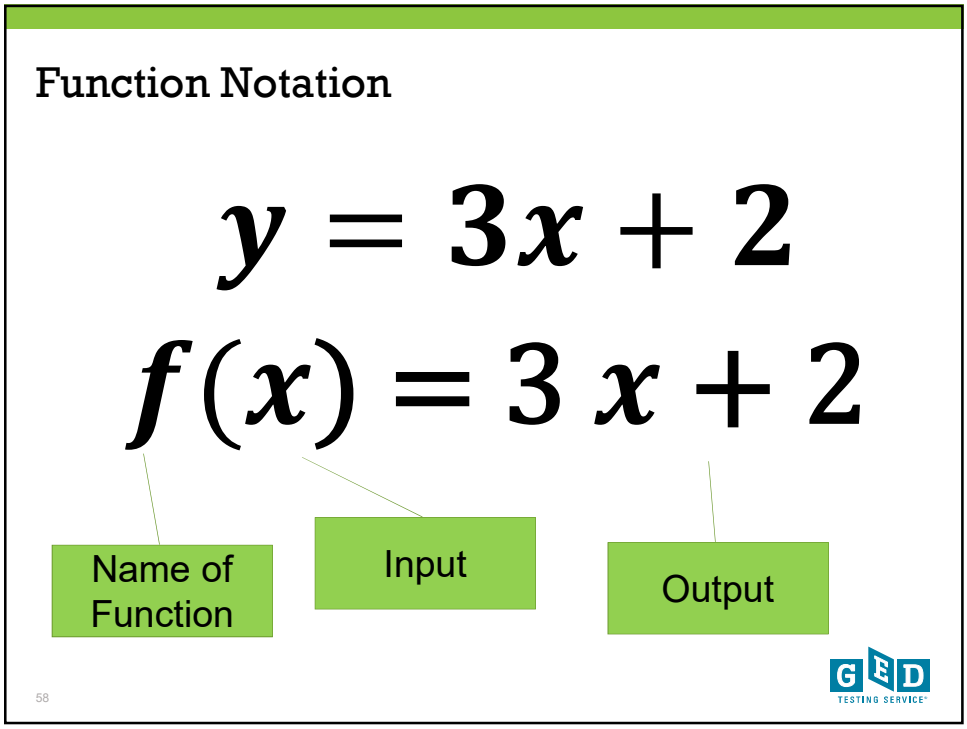
55

Instructions

- Using the sorting mat, categorize the cut-outs into those which represent functions and those that do not.
- Place the function cut-outs inside the circular section of the Venn Diagram.
- Place the non-functions outside the circular section of the Venn Diagram.

56





Evaluate the Function

To find f(-2) you need to substitute a -2 for every *x* value. Then carefully simplify using the order of operations.

Find f (-2).

$$f(x) = 2x^2 - 3x + 6$$

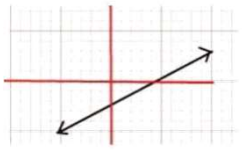
$$f(-2) = 2(-2)^2 - 3(-2) + 6$$

$$f(-2) = 2(4) - 3(-2) + 6 = 8 + 6 + 6 = 20$$

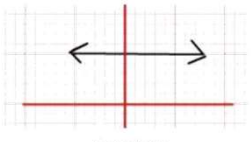
59

Types of Functions – The Next Step

Linear

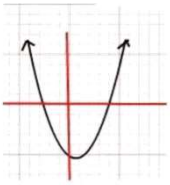


Linear

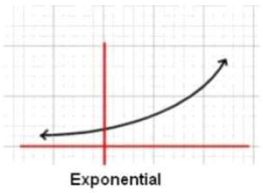


Constant

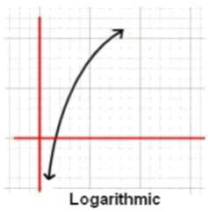
Non-Linear



Quadratic



Exponential



Logarithmic

60

A Real-World Linear Function

A lawyer charges a base (one time) fee of $200 and $75 each hour for consulting with her. Calculate the total cost of the lawyer if you consulted with her for one, two, three, four, or five hours.

61

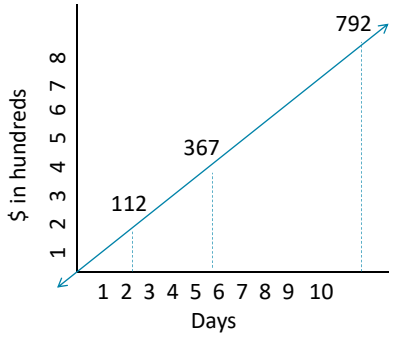
Remember Hartley?

a) $m = \frac{367-112}{5-2} = \frac{255}{3} = 85$

b = 367 – 85(5) = -58

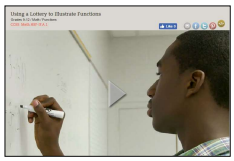
b) y = 85x – 58

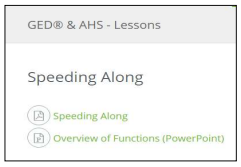
c) y = 85(10) – 58 = 850 – 58 = $792

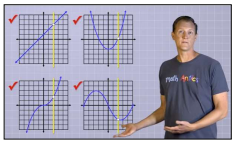


62

Resources – Beginning Looks



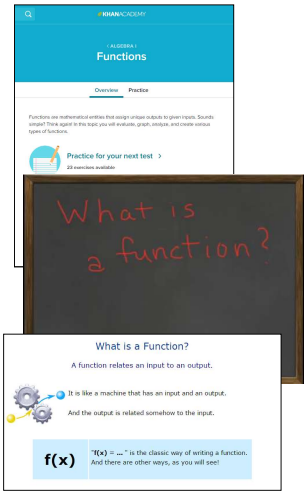




- Using a Lottery to Illustrate Functions - The Teaching Channel
  https://www.teachingchannel.org/videos/teaching-functions?utm_source=Alpha+List&utm_campaign=17fa2b7690-
- Speeding Along
  http://www.floridaipdae.org/index.cfm?fuseaction=resources.GEDAHS&cagiid=A37BC967EEFD18737E7AC2AF2D8421DD4A11C694934330A61EB65F4EB10E766B
- What Are Functions? Math Antics
  https://www.youtube.com/watch?v=52tpYl2tTqk

63

Resources



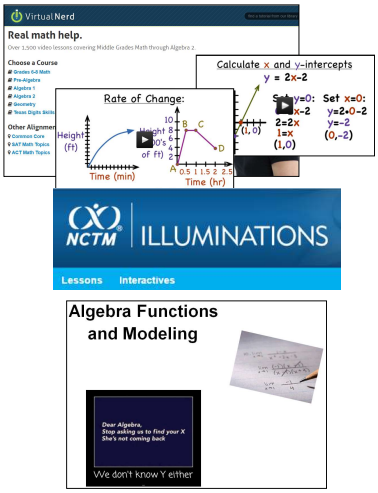
Functions – Khan Academy
https://www.khanacademy.org/math/algebra/algebra-functions

What is a function?
https://www.youtube.com/watch?v=ryQJa8ybxVY

Math is Fun
https://www.mathsisfun.com/sets/function.html

64

Resources



Virtual Nerd
http://www.virtualnerd.com/

Illuminations
https://illuminations.nctm.org/

Algebraic Functions and Modeling – Steve Schmidt, Appalachian State
https://abspd.appstate.edu/node/385

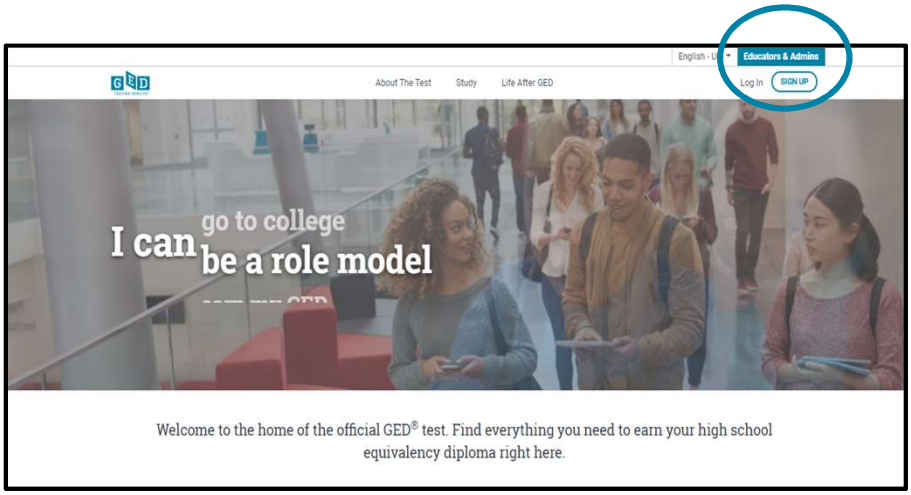
65

Tips for Teaching Inequalities and Functions

- Make it meaningful - start with concrete examples and real-world problems
- Make your thinking processes visible
- Solve the problems many ways
- Show the application
- Provide time for discourse - have students communicate their reasoning
- Ensure time for mastery of the basics

66

https://ged.com



67

GED TESTING SERVICE®

Q & A

68