



Mathematical Reasoning

Transitioning Students from ABE to GED® Level Skills

Facilitator

Ronald Cruz

Coordinator, CARIBE Refugee Program

Mathematics/Physics Teacher

State Trainer Florida IPDAE

National Trainer, GED Testing Service

rcruz@floridaipdae.org

ronaldallan.cruz@sdhc.k12.fl.us

GEDtesting.com • GED.com



Session Objectives



- Discuss Performance Level Descriptors (PLDs) at Levels 1 and 2
- Identify selected skill sets students need to successfully transition from ABE to GED® preparation
- Explore resources to aid students in developing mathematical reasoning skills

GEDtesting.com • GED.com



Understanding Skills Students Have

Low Intermediate Basic Education (4-5.9 GLE)	High Intermediate Basic Education (6-8.9 GLE)	Low Adult Secondary Education (9-10.9 GLE)
Students can perform with high accuracy all four basic math operations using whole numbers up to three digits and can identify and use all basic mathematical symbols.	Students can perform all four basic math operations with whole numbers and fractions; can determine correct math operations for solving narrative math problems and can convert fractions to decimals and decimals to fractions; and can perform basic operations on fractions.	Students can perform all basic math functions with whole numbers, decimals, and fractions; can interpret and solve simple algebraic equations, tables, and graphs and can develop own tables and graphs; and can use math in business transactions.

4 GEDtesting.com • GED.com



Where are the challenges?

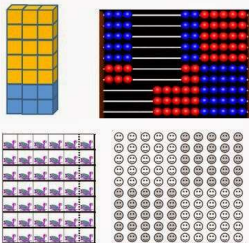
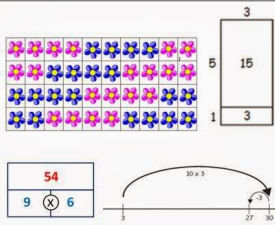
Students at the Adult Basic Education level

- Have *limited* but *developing* proficiency
- Lack consistency in applying skills
- Need to strengthen foundational skills
- Need to develop additional skills

5 GEDtesting.com • GED.com



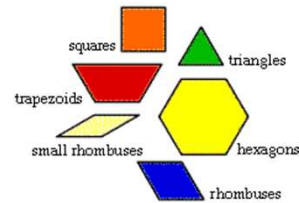
C-R-A – Essential for Understanding

Concrete	Representational	Abstract															
Students manipulate hands-on, concrete materials	Students draw and observe diagrams, or watch the teacher touching and moving hands-on materials	Numbers and mathematical symbols															
		<table><tr><th colspan="5">x 4 Patterns</th></tr><tr><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td></tr><tr><td>24</td><td>28</td><td>32</td><td>36</td><td>40</td></tr></table> 8×5 $(4 \times 2) \times 5$ $4 \times (2 \times 5)$ 4×10 40 $45 \div 5$ $(50-5) \div 5$ $(50 \div 5) - (5 \div 5)$ $10-1$ 9	x 4 Patterns					4	8	12	16	20	24	28	32	36	40
x 4 Patterns																	
4	8	12	16	20													
24	28	32	36	40													

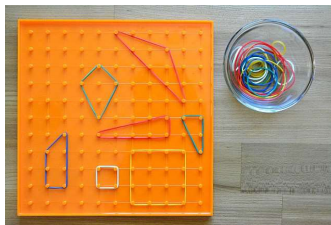
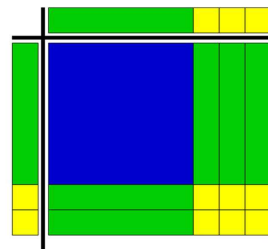
6 GEDtesting.com • GED.com



Best Practice for Building Skills



$$(X+2)(X+3) = X^2 + 5X + 6$$



7 GEDtesting.com • GED.com



Performance Level Descriptors

Focusing Instruction – Moving from
Level 1 to Level 2

GEDtesting.com • GED.com



Skills Typical of Level 1: “Limited and Inconsistent”

Quantitative Problem Solving with Rational Numbers

- Apply number properties involving multiples and factors at a limited and inconsistent level
- Solve real-world problems using rational numbers at a limited and inconsistent level
- Compute unit rates at a limited and inconsistent level

Quantitative Problem Solving in Measurement

- Compute the area and perimeter of triangles and rectangles at a limited and inconsistent level
- Determine side lengths of triangles and rectangles when given area or perimeter at a limited and inconsistent level
- Represent, display, and interpret categorical data in circle and bar graphs
- Represent, display, and interpret categorical data in tables and scatter plots

Algebraic Problem Solving with Expressions and Equations

- Evaluate linear expressions
- Write linear expressions to represent context at a limited and inconsistent level
- Evaluate polynomial expressions at a limited and inconsistent level
- Write rational expressions to represent context at a limited and inconsistent level
- Solve real-world problems involving linear equations at a limited and inconsistent level
- Solve algebraic and real-world problems involving systems of equations

9

GEDtesting.com • GED.com



Level 1 Skills

- Algebraic Problem Solving with Graphs and Functions
 - Locate and plot points in the coordinate plan
 - Interpret unit rate as the slope in a proportional relationship at a limited and inconsistent level
 - For a linear or nonlinear relationship, sketch graphs and interpret key features of graphs and tables in terms of quantities
 - Compare two different proportional relationships, each represented in different ways, at a limited and inconsistent level
 - Represent or identify a function in a table or graph as having exactly one output for each input at a limited and inconsistent level
 - Evaluate linear and quadratic functions at a limited and inconsistent level

10

GEDtesting.com • GED.com



Needed for Level 2 . . . “Consistency”

- Foundational skills identified in Level 1 need strengthening plus additional skills need to be added
- Additional skills are summarized at the end of Level 1
- Among the additions...ordering fractions and decimals; squares and square roots; area and perimeter of polygons; factors; and proportions

11 GEDtesting.com • GED.com



What Are the Foundational ("Must-Haves") in Mathematical Reasoning?

12 GEDtesting.com • GED.com



The Key Skills

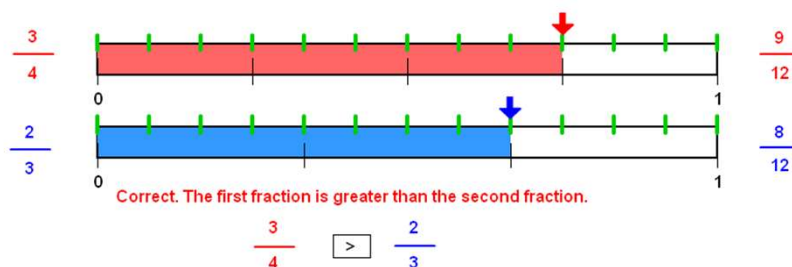
- Consistency
- Fluency
- Number sense
- Measurement with geometric figures
- Working in the coordinate plane
- Interpreting graphics
- Mean, median, and mode

13 GEDtesting.com • GED.com



Ensure Students Can Use a Number Line

The fractions $\frac{3}{4}$ and $\frac{2}{3}$ are pictured with number lines below:



14 GEDtesting.com • GED.com



How Would You Teach It?

- Determine how you would teach this skill
 - Introduce this topic
 - Model your approach to a solution by making your thinking process visible
- Highlight two or three points from your script
- Let's discuss

Instructions: Order the original numbers provided below in ascending order using a number line.

- $\frac{5}{8}$
- 0.8314
- $\frac{1}{16}$
- 0.4823
- $\frac{5}{12}$

15 GEDtesting.com • GED.com



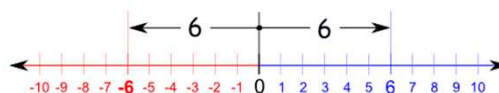
Check Students' Understanding of "Absolute Value"

Absolute Value means how far a number is from 0.

- Remove any negative sign and think of all numbers as positive
- Recognize symbol used to represent absolute value

$$|-5| = 5$$

$$|7| = 7$$



"6" is 6 away from zero,
and "-6" is **also** 6 away from zero.

So the absolute value of 6 is 6,
and the absolute value of -6 is also 6

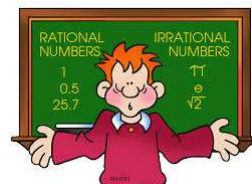
16 GEDtesting.com



Execute Basic Operations on Rational Numbers

Test-Takers should be able to:

- Multiply and divide with decimals
- Compute
 - With fractions, mixed numbers, and negative numbers
 - Using order of operations



17 GEDtesting.service.com • GED.com



Order Matters

Here is your problem: $4 + 2 \times 3 =$

Is the answer 18 or 10?

- Avoid confusion in how problems are solved
- Set up rules of precedence or rank of operations
- Is critical to simplifying and solving different algebra problems

18 GEDtesting.service.com • GED.com



Address Misconceptions about Order of Operations

Misconception 1 - All multiplication should happen before division.

Incorrect	Correct
$12 \div 3 \times 4$	$12 \div 3 \times 4$
$12 \div 12$	4×4
1	16

Parenthesis
Exponents
Multiply / **D**ivide
Add + **S**ubtract

Misconception 2 – All addition comes before subtraction.

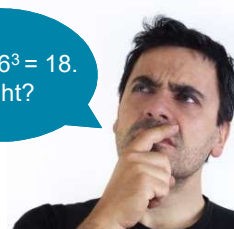
Incorrect	Correct
$4 + 10 - 5 + 8$	$4 + 10 - 5 + 8$
$14 - 13$	$14 - 5 + 8$
1	$9 + 8$
	17

Remember: M/D have the same precedence. Evaluate as they appear from left to right. Same with A/S.

A Continuing Problem

Students think an exponent is the same as a multiplier.

I know $6^3 = 18$.
Right?



Multiplication =
Repeated Addition

$$6 \times 3 = 18$$

$$6 + 6 + 6 = 18$$

Exponents = Repeated
Multiplication

$$6^3 = 18$$

$$6 \times 6 \times 6 = 216$$

Rules of Exponents

	Rule	Example
1	$x^1 = x$	$5^1 = 5$
2	$x^0 = 1$	$5^0 = 1$
3	$x^{-1} = \frac{1}{x^1}$	$5^{-1} = \frac{1}{5}$
4	$(x^m)(x^n) = x^{m+n}$	$(x^2)(x^3) = x^{2+3} = x^5$
5	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^3}{x^2} = x^{3-2} = x^1$
6	$(x^m)^n = x^{(m)(n)}$	$(x^3)^2 = x^{(3)(2)} = x^6$
7	$(xy)^n = x^ny^n$	$(xy)^3 = x^3y^3$
8	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$
9	$x^{-n} = \frac{1}{x^n}$	$x^{-2} = \frac{1}{x^2}$

21 GEDTestingService.com • GED.com



Squares and Square Roots of Positive Rational Numbers

Have students...

- Memorize the first 12 perfect squares (1, 4, 9, . . . 144)
- Understand inverse relationship between pairs of squares and square roots ($12^2 = 144$ and $\sqrt{144} = 12$)
- Understand squaring a negative number vs the negative of a squared number, e.g. $(-3)^2 = 9$ and $-(-3)^2 = -9$
- Practice computing with squares and square roots that include fractions and decimals



22 GEDTestingService.com • GED.com

Cubes and Cube Roots of Rational Numbers

Have students...

- Memorize the first 6 perfect cubes (1, 8, 27, . . . , 216)
- Understand the following:
 - Inverse relationships between cubes and cube roots
 - Cubing a negative number versus the negative of a cubed number
 - Practice computing with cubes and cube roots that include fractions and decimals



23 GEDtestingservice.com • GED.com

Undefined Value Over the Set of Real Numbers

- Reinforce skills on questions that involve
 - Zero in the denominator
 - Fractions with expressions equivalent to zero in the denominator
 - Square roots of negative numbers
 - Expressions that when simplified result in square roots of negative numbers
 - Substitution with linear expressions

24 GEDtestingservice.com • GED.com



Calculator-Prohibited Indicators

TEST-TAKER RECOMMENDATIONS FOR CALCULATOR-PROHIBITED INDICATORS		
<p>The first several questions of the GED® Mathematical Reasoning test assess eight indicators covering various concepts in number sense and computation (Q.1.a through Q.1.d and Q.2.a through Q.2.d) that prohibit the use of the calculator. GED Testing Service has analyzed data on these calculator-prohibited items, resulting in the following comments and recommendations:</p>		
Indicator	Background	Recommendations for Test-takers
Q.1.a Order fractions and decimals, including on a number line.	<p>These questions may require</p> <ul style="list-style-type: none"> comparing or ordering positive numbers, or negative numbers, or both, with or without a number line. <p>Test takers generally do very well on this indicator, with the exception of questions that require test takers to compare or order a set consisting entirely of negative numbers.</p>	<ul style="list-style-type: none"> Leverage skills in comparing and ordering positive fractions and decimals toward similar skills comparing and ordering negative fractions and decimals. Understand the difference in how negative numbers are compared and ordered: <ul style="list-style-type: none"> For instance, while 0.7 is greater than 0.2, -0.7 is actually less than -0.2. Since positives and negatives are essentially opposites, the rules for ordering each type of number are applied in a similarly opposite manner.
Q.1.b Apply number properties involving multiples and factors, such as using the least common multiple, greatest common factor, or distributive property to rewrite numeric expressions.	<p>Test takers generally perform very well on this indicator, which includes questions that include both context or pure computation (no context) and which test factors of a number, multiples of a number, least common multiple, greatest common factor, etc.</p>	<p>No specific recommendations are provided, as the general population of GED® test takers performs well on this indicator.</p>



Sample Items

- Ordering Fractions and Decimals** Place the following numbers in order from greatest to least: 0.2, -1/2, 0.6, 1/3, 1, 0, 1/6
- Factors and Multiples** Find the LCM that is necessary to perform the indicated operation. $7/6 - 1/4 =$
- Rules of Exponents** Simplify the following: $(x^3)^5$
- Distance on a Number Line** Find the distance between two points -9 and -3 on a number line.

Sample Items

- Operations on Rational Numbers Solve: $3 \frac{1}{2} \div 3 \frac{1}{2} =$
- Squares and Square Roots of Positive Rational Numbers Find $\sqrt{9}$ Find $\sqrt{24}$
- Cubes and Cube Roots of Rational Numbers Find $(-4)^3$
- Undefined Value Over the Set of Real Numbers Solve $(2x - 3)(x + 2) = 0$

27 GEDtesting.com • GED.com



Reading + Reasoning in Mathematics

Steps to Success

28 GEDtesting.com • GED.com



We Need to Help Students . . .

 Formula Sheet

Build their
reading skills

 Calculator Reference

Type your answer in the box. You may use numbers, a decimal point (.), and/or a negative sign (-) in your answer.

Hartley opened a food truck business to sell food on the street. On day 2, the business earned \$112. On day 5, the business earned \$367. Hartley assumes that the earnings will continue to increase at the same rate. How much will the business earn on day 10?

\$

Build math
vocabulary

Depend less on
rote memory

Increase conceptual
knowledge

29 GEDtestingservice.com • GED.com



Two Essential Strategies

Helping students learn *how to learn* is critical to guiding the development of higher-order thinking skills.

- **Modeling** is a way to teach students how to learn (by having them observe higher-order thinking as it occurs and allowing them to emulate it)
- **Scaffolding** allows students to practice with diminishing support—to build competence and confidence

30 GEDtestingservice.com • GED.com



Reading and Reasoning Process

First Read: Read for Understanding

Second Read: Identify a Problem-Solving Process

Third Read: Solve the Problem and Check for Reasonableness

Miller, P. and Koesling, D. "Mathematics Teaching for Understanding: Reasoning, Reading, and Formative Assessment." Danvers, MA

31 GEDtestingservice.com • GED.com



First Read: Read for Understanding

- Read through the problem aloud – noting your reactions to what you're reading.
- What vocabulary do you not know?
- What's the real-world context of the problem?
- Is there a picture that can help you visualize the problem?
- What questions are being asked?

Miller, P. and Koesling, D. "Mathematics Teaching for Understanding: Reasoning, Reading, and Formative Assessment." Danvers, MA

32 GEDtestingservice.com • GED.com



First Read: Read for Understanding



An apartment building contains 12 units consisting of one- and two-bedroom apartments that rent for \$360 and \$450 per month, respectively. When all units are rented, the total monthly rental is \$4,950. What is the number of two-bedroom apartments?



33 GEDtesting.com • GED.com

Second Read: Identify a Problem-Solving Process

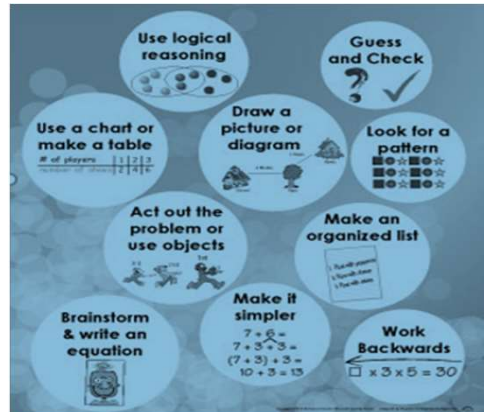
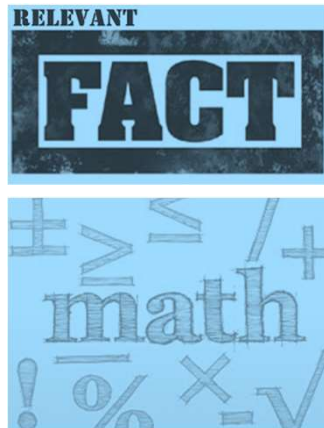
- What is the pertinent information in this problem?
- What problem-solving strategies could I use?
- Which of those problem-solving strategies is best suited for this problem?
- How will I represent the problem in the symbolic language of mathematics?
- What mathematical details will I select as I reason and solve this problem?

Miller, P. and Koesling, D. "Mathematics Teaching for Understanding: Reasoning, Reading, and Formative Assessment." Danvers, MA

34 GEDtesting.com • GED.com



Second Read: Identify a Problem-Solving Process



Miller, P. and Koesling, D. "Mathematics Teaching for Understanding: Reasoning, Reading, and Formative Assessment." Danvers, MA

35 GEDtesting.com • GED.com



Begin with the Basics

Noticing

- Allows all students to participate
- Work independently or in groups
- Focuses on what is stated in the problem
- Identifies what are the "givens" of the problem

Wondering

- Is the planning part
 - Talk about strategies to use
 - Restate the problem
 - Pose questions about what is noticed
 - Supports students slowing down and thinking
 - Prompts brainstorming, listing, and discussing ideas

36 GEDtesting.com • GED.com



Third Read: Solve the Problem and Check for Reasonableness

- Now that I understand the problem's content, how can I best use my math skills to solve the problem?
- Am I answering the right question?
- How should the answer to the question be expressed?

Miller, P. and Koesling, D. "Mathematics Teaching for Understanding: Reasoning, Reading, and Formative Assessment." Danvers, MA

37 GEDtesting.com • GED.com



Routines for Problem Solving Build Reasoning Skills

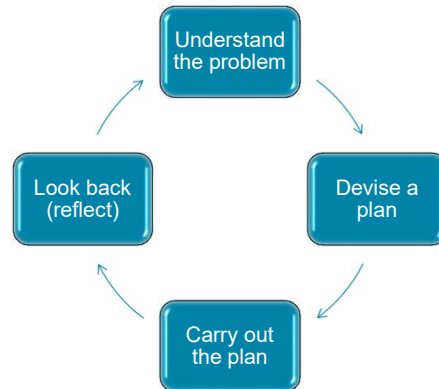


38 GEDtesting.com • GED.com



Solution Pathways = Problem Solving

Polya's Four Steps to Problem Solving



Polya, George. *How To Solve It*, 2nd ed. (1957). Princeton University Press.

39 GEDtestingservice.com • GED.com



The Value of Teaching with Problems



- Places students' attention on mathematical ideas
- Develops "mathematical power"
- Develops students' beliefs that they are capable of doing mathematics and that it makes sense
- Provides ongoing assessment data
- Allows an entry point for a wide range of students

40 GEDtestingservice.com • GED.com

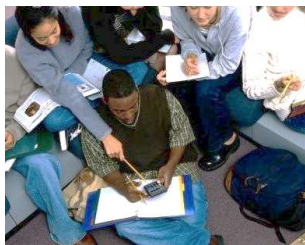


Problem Solving In the Classroom

What opportunities do your students currently have to grapple with non-routine complex tasks and...



.... to reflect on their thinking and consolidate new mathematical ideas and problem solving solutions?



41 GEDtestingservice.com • GED.com



Focus on...

- Providing ample practice in the basics to ensure consistency
- Increasing emphasis on geometric reasoning
- Shifting from the “rules or processes” of mathematics to deepening the understanding of “why”
- Helping students learn how to translate from words to symbols
- Having high expectations of all students

42 GEDtestingservice.com • GED.com



Resources

43 GEDtesting.com • GED.com



Resources – A Few to Get Started

Building a Number Line

<https://unctv.pbslearningmedia.org/resource/mgbh.math.ns.numblne/building-a-number-line/#.WU1B-IWcHnM>

Answer the Why – Order of Operations

<https://www.khanacademy.org/math/pre-algebra/pre-algebra-arith-prop/pre-algebra-order-of-operations/v/introduction-to-order-of-operations>

Rules of Exponents – The Math Dude

<http://www.montgomeryschoolsmd.org/departments/itv/MathDude/watch-online.aspx?id=22>

Math is Fun – Properties of Zero

<http://www.mathsisfun.com/numbers/zero.html>

National Library of Virtual Manipulatives for Math

<http://nlvm.usu.edu/en/nav/index.html>

44 GEDtesting.com • GED.com




www.GEDtestingservice.com









Professional Development



EDUCATORS
POLICYMAKERS
MEDIA
PUBLIC


SCHEDULE NOW ON
MyGED™

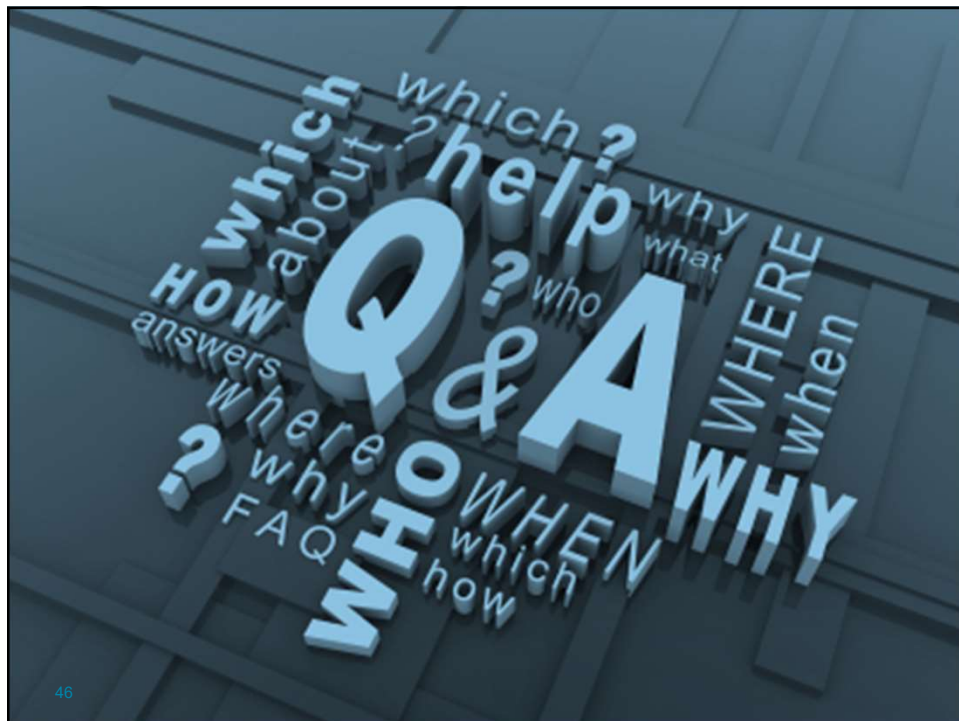
GET YOUR
TRANSCRIPT


CANADIAN


INTERNATIONAL

ABOUT GED TESTING SERVICE | ABOUT THE GED® TEST | PRIVACY POLICY





Thank you!

Communicate with GEDTS

communications@GEDTestingService.com

Ronald Cruz

Hillsborough County Public Schools, FL

Institute of Professional Development of Adult

Educators (IPDAE)

rcruz@floridaipdae.org

ronaldallan.cruz@sdhc.k12.fl.us

47 GEDTestingService.com • GED.com

